Motivation

Optical blurring of an image is the result of light rays from a single object focusing at a point before or after the sensor in a camera. This physical situation can be described as a convolution of the true captured image with an optical filter: the point spread function (PSF).
Equation:

$$i \qquad t \quad k \quad t \; f \qquad t$$

Given that the PSF works much like a frequency response for a spatial domain, we explore the possibility of extracting a focused image from an image that has been captured out of focus.
Equation:

$$D \quad K \quad I$$

where D, K, I are the transforms of the deblurred image, the PSF, and a blurred image.

Instead of using the full circular aperture of most cameras, we place a coded mask within the lens of our camera. The mask itself is nothing more than a binary-coded transparency, but it enables us to force a particular PSF into our camera system. By recreating the mask's code in software, we can recover the PSF that is appropriate for the depth at which our image would have been focused. Once we have the appropriate PSF for a particular focus-depth (which correlates to increasing or decreasing the size of the mask in software), we use the Richardson-Lucy Deconvolution algorithm to recover a focused image.

Mask Finding and Scaling

The PSF is effectively the impulse response in the optical domain. If we try to describe a system with an incorrect impulse response, the output is not true. Accordingly, every plane at a particular depth has its own impulse response, or PSF in this case. The size or scale of a PSF corresponds to a particular plane of focus in space and a blurred image is effectively captured when a lens is focused at a depth that does not correspond to the plane in which the object exists. This is the same situation we recreate when we capture our blurred images.

For these reasons it is important that we obtain not only an exact replica of the PSF used to capture the blurred image, but also the exact scale for the depth of the captured image. An incorrect PSF yields less than desirable results and after experimenting with various techniques we developed two methods for generating the PSF for a given mask and depth in MATLAB: manual construction of the matrix and a search algorithm.

Hand Mask

```
0       0
0       1
```



```
0       0       0       0
0       0       0       0
0       0       1       1
0       0       1       1
```

Original Matrix of Scale 1 is transformed to a Matrix of Scale 2.

When the distance from the camera to the object is unknown, manual construction of a mask is preferred. The small size of this method allows

the mask to be scaled easily as shown above so that the appropriate PSF for a given distance can be searched. The masks are represented by an N-by-N matrix, where every black block is marked as a 0 and every transparent block is marked as a 1.

The simplicity of this method is appealing and provides decent results. One of the main drawbacks to creating the matrix manually is that the actual mask hardly ever appears exactly in the image. For example, the corners of our masks are rounded by the camera aperture. We are able to compensate for this by reducing the intensity of the corner entries in our matrix to about 0.5. A second drawback is that rotations in the placement of the mask in the camera reduce the effectiveness of our manually constructed matrix.

Auto Mask
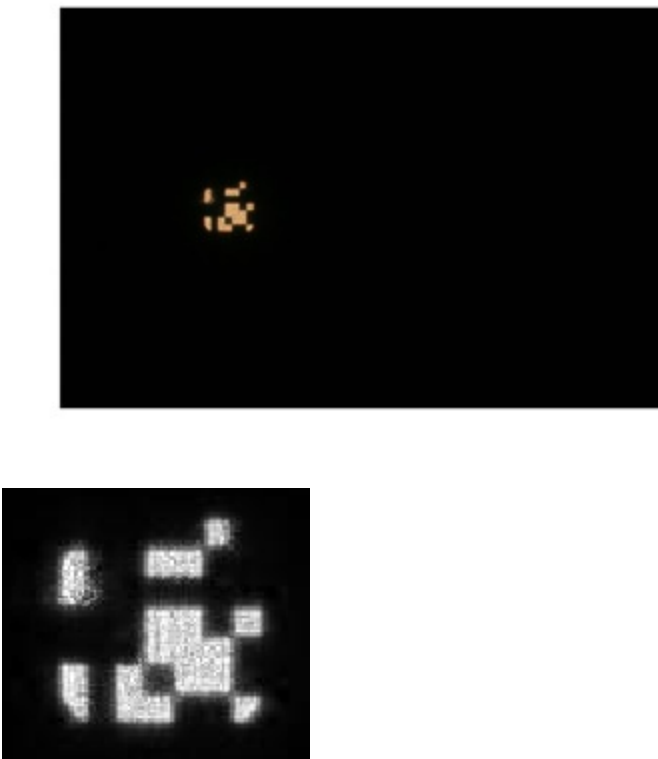
Captured PSF (Left) and resulting mask (right) using Automask.

After capturing an image of the appropriate PSF for the depth at which our object exists, we isolate the PSF in MATLAB. First the PSF image is

converted to black and white. Given the simplicity of our algorithm, it is necessary to apply some smoothing to this black and white image. After, it is cropped and reproduced automatically using a 0-1 search algorithm, yielding a high-resolution matrix of the appropriate PSF. This method eliminates the need for scaling and much of the rigor required to create the PSF manually, such as aligning the mask exactly. Overall the mask that is recovered by this method is truer to the actual mask and results in better deblurring.

Then What?

Once a PSF is obtained, either by Hand Mask or Auto Mask, we can deblur using the blurred image, the PSF, and the Richardson-Lucy Deconvolution Algorithm.

Richardson-Lucy Deconvolution

We used the Richardson-Lucy algorithm to recover our true image that has been blurred by our point spread function. Our observed image di is the sum of our point spread function, which is the fraction of light observed at position *i* originating from true location j) multiplied by the pixel value uj of the true image. Photon noise is Poisson distributed, therefore the Richardson-Lucy algorithm assumes that all uj pixel values are Poisson distributed.

We can iteratively solve for the most likely uj according to the formula:

$$d_i = \sum_j p_{ij} u_j$$

$$u_j^{(t+1)} = u_j^{(t)} \sum_i \frac{d_i}{c_i} p_{ij}$$

$$c_i = \sum_j p_{ij} u_j^{(t)}.$$

Because our masks are complicated, Richardson-Lucy will create spatial ringing since we have multiple locations in which light can be transmitted through and it does not always correctly determine the origin of that particular ray. This is why the pinhole aperture deblurred image contains no ringing artifacts because the Richardson-Lucy algorithm will correctly find the true image as the single point because that is always the maximum.

Recovering Intensity
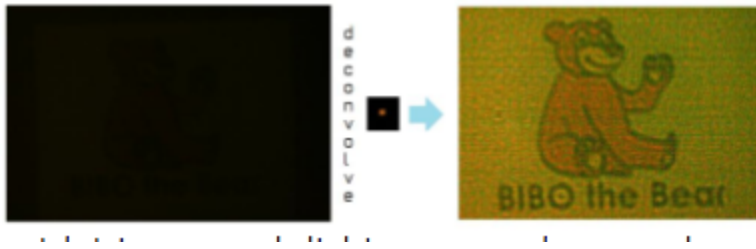
Image captured using a pinhole mask (left) and the deblurred image (right) without light correction.

The mask reduces the amount of light that enters the camera aperture, reducing the brightness of the captured images. The deblurred image we recover is dim compared to the same image captured without the coded aperture so we compensate by scaling the intensity of the deblurred image according to the ratio of opaque area in the mask to the total area of the mask. The intensity of the deconvolved image is extracted, scaled, then replaced for a more appealing image.

Results and Remarks

Scale (7x7 Matrix Size to original PSF)	Signal to Noise Ratio
2 <- Scale our code found	20.8281 <- MAX SNR
3	20.7118
4	20.6709
5	20.6521
6	20.6418
7	20.6356
8	20.6314
9	20.6286

Visually our deblurred images appear to have good results, but SNR measurements with a clear natural image show that we are effectively deblurring at the correct scale.

Conclusions and Suggestions

As can be seen from the images above, our efforts produce qualitative and quantitative results. Clearly the image of the bear shows significant deblurring and the SNR supports that our methods have chosen the appropriate PSF to do so.

The SNR table shows that there are small differences between scales. This is a result of some downsampling we used to simplify the analysis process and should not suggest that any arbitrary scale may be used to deblur. In reality, there is a meaningful difference in the SNRs of various scales.

Finally, we would like to suggest an extension to our project. We were only able to successfully deblur the images of objects that existed in a flat plane, at one depth, in other words. The applications of this method could be extended significantly by developing a way to characterize the different depths of a real life image. A depth-map would provide the means to use the correct PSF for every point in the image, producing an all-focus image.